

Nabi, S. W., and Vanderbauwhede, W. Using Type Transformations to Generate Program Variants for FPGA Design Space Exploration. In: 2015 International Conference on ReConFigurable Computing and FPGAs (ReConFig), Mexico City, Mexico, 7-9 Dec 2015, pp. 1-6. ISBN 9781467394055.

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Deposited on: 09 March 2016

# Using Type Transformations to Generate Program Variants for FPGA Design Space Exploration

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**Abstract**—We present preliminary results with the TyTra design flow. Our aim is to create a parallelising compiler for high-performance scientific code on heterogeneous platforms, with a focus on Field-Programmable Gate Arrays (FPGAs). Using the functional language *Idris*, we show how this programming paradigm facilitates generation of different correct-by-construction program variants through *type transformations*. We have developed a custom Intermediate Representation (IR) language, the TyTra-IR, which is similar to the LLVM IR, with extensions to express parallelism, allowing us to design variants associated with each program variant. The key innovation of the TyTra-IR is the ability to construct and cost design variants for FPGAs. Our prototype compiler generates Verilog code for FPGA synthesis from a given IR description. Using a real-world Successive Over-Relaxation (SOR) kernel, we illustrate generation of program variants in *Idris*, their representation in TyTra-IR, and evaluation of variants using our cost-model. We compare the estimates from the cost-model with results from synthesis and simulation of equivalent HDL.

## I. INTRODUCTION

Platforms for High-Performance Computing (HPC) are becoming increasingly heterogeneous with the adoption of GPUs, many-core accelerators and FPGAs. Such platforms present a significant programming challenge, especially because the key users of HPC resources are scientists, not parallel programmers. The work we present in this paper aims to facilitate the use of Field-Programmable Gate Arrays (FPGAs). These devices are very promising in terms of energy efficiency, but programming them presents a major obstacle to their wider adoption in HPC. High-level programming tools for FPGAs have made significant contributions, but require considerable effort to find the best design variant from the complex design space offered by the fine-grained reconfigurability of FPGAs.

Our main contribution is raising the programming abstraction for FPGAs such that we can express the design in a functional language like *Idris* as shown in Figure 1. This functional abstraction enables *type transformation* that *reshape* the data to create a variants that are *correct-by-construction*. The transformation implies a reconfigured FPGA architecture for that reshaped data, effectively creating a new design variant. A light-weight cost-model for evaluation of multiple design variants opens the route to a fully automated compiler that can: generate variants, evaluate them, choose the best option, and generate HDL code for it.

Multiple program variants are generated using *type-transformations* that are intrinsically *safe*. Each program vari-

ant in *Idris* is associated with a design variant in a lower-level description using an IR, the *Tytra-IR*. The TyTra Back-End Compiler (TyBEC) then generates resource-utilization and performance estimates, and emits Hardware-Description Language (HDL) code in Verilog for the chosen program variant. Currently we translate the *Idris* code to TyTra-IR manually, and a front-end compiler for doing this is a work in progress. The back-end compiler can cost the IR code and emit Verilog code.

Our running exemplar is a real kernel from the scientific computing domain, the successive over-relaxation (SOR) kernel used in a weather model [1] through which we illustrate generation of program variants in *Idris*, their representation in TyTra-IR, and evaluation of variants using our cost-model.

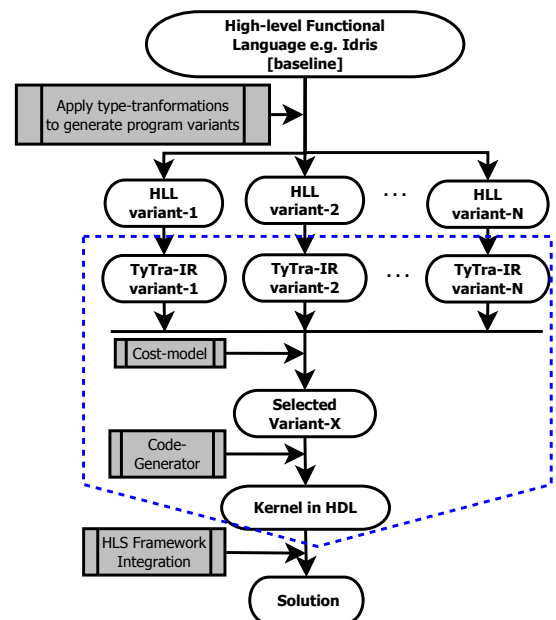


Fig. 1. Design entry in the TyTra flow is in a functional language like *Idris*, variants are generated using type-transformations and converted to the TyTra-IR. The back-end compiler costs the variants and emits HDL code, which can be integrated with an HLS tool for working solution. The dotted line marks the stages in the flow that are currently automated.

## II. PROGRAM GENERATION THROUGH TYPE TRANSFORMATIONS

We aim to demonstrate how a program can be rewritten in a high-level language that facilitates generation of differ-

ent, correct-by-construction instances of that program through type- transformations. Each program variant will have a different performance related to its degree and type of parallelism, and a different cost. Through our cost model we are able to select the best suited instance in a guided optimisation search.

#### A. Exemplar: Successive Over-Relaxation

Consider the following SOR kernel, taken verbatim from the code for the Large Eddy Simulator, an experimental weather simulator [1] written in Fortran.

```
do l=1,nmaxp ; do k=1,km ; do j=1,jm ; do i=1,im
  reltmp = omega*(cn1(i,j,k)*
    (cn2l(i)*p(i+1,j,k)+cn2s(i)*p(i-1,j,k) &
    +cn3l(j)*p(i,j+1,k)+cn3s(j)*p(i,j-1,k) &
    +cn4l(k)*p(i,j,k+1)+cn4s(k)*p(i,j,k-1) &
    -rhs(i,j,k))-p(i,j,k))
  p(i,j,k) = p(i,j,k) + reltmp
  sor_err = sor_err + reltmp*reltmp
end do ; end do ; end do ; end do
```

The kernel iteratively solves the Poisson equation for the pressure assuming periodic boundary conditions in the transverse ( $i$ ) direction and open (Neumann) conditions in other directions. The main computations are a stencil over the neighbouring cells (which is inherently parallel), and a reduction to compute the remaining error. The boundary conditions are simple copy operations. As the boundary conditions do not involve any computation, they are not discussed in what follows. The SOR algorithm is iterative, when the error is smaller than a preset value or when a maximum number of steps is reached, the algorithm stops.

#### B. Idris, Higher-Order Functions *map* and *fold*, and Dependent Types

Functional languages can express higher-order functions i.e. functions that take functions as arguments and can return functions. They support *partial application* of a function, and have strong *type safety*. These features make them suitable as a high-level design-entry point, and for generating correct-by-construction or *safe* program variants through type transformations. We have chosen the *Idris* language [2] because it is *dependently typed*, which allows the size of the data to be expressed explicitly in the type. This feature is crucial for our purpose of generating program variants by reshaping data and ensuring correctness through type safety.

To describe computations on finite-size ordered sets of data we use a dependent vector type which encodes the size of the vector<sup>1</sup>:

```
p1D : Vect (ip+3)*(jp+3)*(kp+2) Float
```

Here the type of *p1D* is the entire string after `:`, showing it is a vector of size equal to size of the 3D matrix, and of type *Float*. Multi-dimensional vectors are obtained through nesting:

```
p2D : Vect (jp+3)*(kp+2) (Vect ip+3 Float)
```

The main higher order functions we will use are *map* and *fold*, which capture the computation pattern at a higher abstraction than the more familiar *for* loops. The *map* operation

simply applies a function to a vector, similar to a dependency-free *for*-loop. We can write its type as<sup>2</sup>:

```
map : (t1 -> t2) -> (Vect sz t1) -> (Vect sz t2)
```

The *foldl* operation (which performs reduction) applies a function to a vector and an accumulator, similar to a *for*-loop that performs an accumulation. We can write its type similarly as

```
foldl : (t2 -> t1 -> t2) -> t2 -> (Vect sz t1) -> t2
```

These higher order functions allow us to transform programs through transformation of the types, as discussed in §II-D. Although expressing programs in terms of *map* and *fold* might seem restrictive, the paradigm is in fact expressive enough to express most scientific codes and is a very good starting point for stream-based FPGA programming.

#### C. SOR Kernel in Idris

The functions *map* and *fold* perform computations on the vector without explicit iterators. Therefore, to calculate e.g. the SOR expression as above, we need to define a set of vectors, one for every term in the expression. This means that we need a function that will take the original vectors *p*, *rhs*, *cn\** and return a single new vector of size *im.jm.km*, where each element is a tuple consisting of all terms required to compute the SOR, i.e. the pressure at a given point and its 6 neighbouring cardinal points, the weight coefficients *cn\** and the *rhs* term for a given point. The implementation of this function is simply a copy operation and not our main concern.

```
pps = prepare_vectors p rhs cnl cn2l ...
```

Given the new vector of tuples, we can define the actual SOR computation as

```
ps = map p_sor pps
```

where *p\_sor* computes the new value for the pressure for a given input tuple from *pps*:

```
p_sor pt = reltmp + p_c
  where
    (p_i_p1,...,p_c,rhs_c) = pt
    reltmp = omega * (cn1 * (
      cn2l_x * p_i_p1 + cn2s_x * p_i_m1
      + cn3l_x * p_j_p1 + cn3s_x * p_j_m1
      + cn4l_x * p_k_p1 + cn4s_x * p_k_m1 )
      - rhs_c) - p_c
```

#### D. Type Transformations

Our main purpose is to generate many *safe* variants by transforming the *type* of the various functions making up the program and *inferring* the program transformations from the type transformation. The details and proofs of the type transformations and their safety are available in [3]. In brief, we reshape the vector in an order-preserving manner and infer the corresponding program that produces the same result. Each reshaped vector in a program variant translates to a different

<sup>1</sup>Note that in Idris, arguments in a type signature or a function call are separated by a space.

<sup>2</sup>Function types are specified by the type of each argument, separated by the arrow `->`, with the last argument being the return type.

arrangement of streams. We then use our cost-model to choose the best design, as we will discuss in §V-A.

Let's assume that the type of the 1-D *prepared* vector is  $t$  and its size  $im.jm.km$ , which we can turn into e.g. a 2-D vector with sizes  $im.jm$  and  $km$  using type-transformation:

```
pps : Vect (im*jm*km) t      --1D vector
ppst: Vect km (Vect im*jm t) --transformed 2D vector
```

Resulting in a corresponding change in the program:

```
ps = map p_sor pps      --original program
ppst = reshapeTo km pps --reshaping data
pst = map (map p_sor) ppst --new program
```

where *map p\_sor* is an example of partial application. Because *ppst* is a vector of vectors, the outer *map* takes a vector and applies the function *map p\_sor* to this vector.

### E. Parallelism Transformations

As a *map* is by definition a dependency-free operation, it can in principle be executed in parallel on all elements of the vector. It can of course also be executed sequentially, or as a stream. This means that the original program has three variants, one for each type of *map*, i.e. parallel, sequential and pipelined. A transformed program with two nested *maps* has nine variants. Each program transformed by *reshaping* the data will have these nine variants. The number of transformed program types is equal to the number of possible integer divisions of the original type. In this fashion it is possible to generate large numbers of safe program variants. In §IV we discuss the cost-model we developed to evaluate the variants. As part of our future work, we aim to limit the number of program variants generated and evaluated, by developing heuristics for constrained variant generation.

### F. Reductions

The *map* higher-order function is limited to dependency-free vector operations. To express dependencies in a reduction, we can use the *foldl* higher-order function. In order to incorporate the computation for the global SOR error we change the return value of the function *p\_sor* to a tuple that contains two values, i.e. the pressure and the error from that iteration:

```
p_sor pt = (p_c+reltmp, reltmp*reltmp) where ...
```

We apply *map* and separate out the arrays using *unzip* (which transforms a vector of tuples into a tuple of vectors); to obtain the cumulative *sor\_err* we sum *rtsgs*:

```
(ps,rtsgs) = unzip (map p_sor pps)
sor_err = sum rtsgs
```

Here, `sum = foldl (+) 0`. If we transform the type of *pps* as before, the type of *rtsgs* will be transformed and hence the fold operation will be transformed as explained in [3]:

```
sor_err = foldl (foldl (+) 0) rtsgs
```

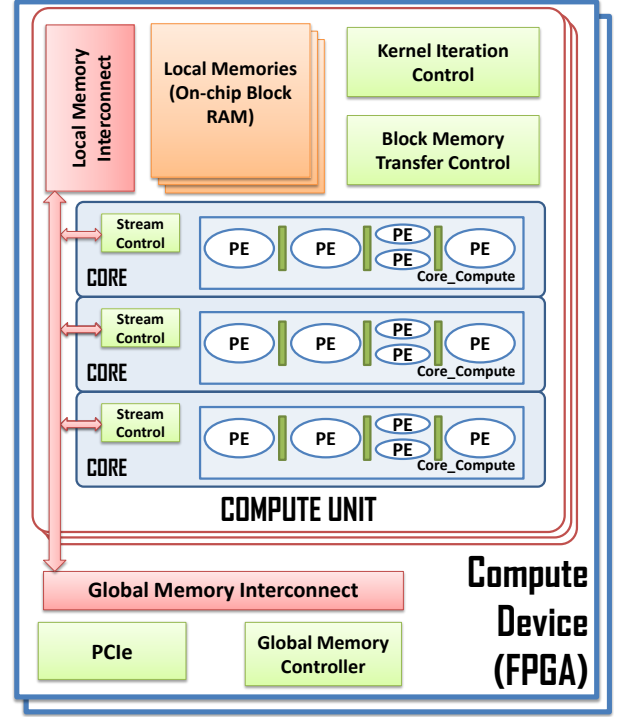


Fig. 2. The TyTra-FPGA platform model abstraction, adapted from the OpenCL platform model.

Furthermore, we could apply an additional type transformation to *rtsgs* as well. It is clear that the number of variants will quickly become very large, but each of them will produce the same result with a potentially different performance and cost.

Now we will discuss the platform model abstraction for the FPGAs that we use, the IR into which variants from Idris can be compiled, and the cost-model used to evaluate the performance and resource utilisation.

## III. THE TYTRA PLATFORM MODEL AND IR LANGUAGE

Our platform model (Figure 2) is based on the generic OpenCL platform model [4]. It is somewhat more nuanced than OpenCL's to incorporate FPGA-specific architectural features, which is similar to how Altera-OpenCL deals with this [5].

The TyTra-IR is a strongly and statically typed language, and all computations are expressed using Static Single Assignments (SSA). It is based on the LLVM-IR, with extensions for parallelism semantics suitable for an FPGA target. This gives us a baseline for designing our language, and will allow us to explore LLVM optimizations, as e.g. the LegUp [6] tool does. The key semantic incompatibility between our IR and LLVM-IR arises due to: first, our IR targeting a *streaming* data-flow architecture, whereas LLVM-IR is designed to be neutral representation for microprocessor targets; and second, our IR having *functions* with added semantics to describe parallelism.

The TyTra-IR code for a design has two components. The *Manage-IR* sets up the streaming data ports for the kernel. It corresponds to the logic in the *core* outside the *core-compute* (See Figure 2). All *Manage-IR* statements are wrapped inside

the `launch()` method. The *Compute-IR* describes the datapath logic. It works with streaming data abstractions. All Compute-IR statements are in the scope of the `main()` function or other functions “called” from it. A detailed discussion of the TyTra-IR syntax is outside the scope of this paper, though an illustration is given for the SOR example in §II-C and a more detailed discussion is available in [7].

#### IV. COMPILER’S COST MODEL AND CODE GENERATOR

We have designed the TyTra-IR specifically to enable estimates of reasonable accuracy, which allow us to evaluate design variants. Our compiler can calculate these estimates directly from the IR without any further synthesis: the throughput estimate, and the resource utilization for a specific Altera FPGA device (ALUTs, REGs, Block-RAM, DSPs). It can also emit synthesizable Verilog code. Figure 3 illustrates the steps taken by our compiler for cost-estimation and code-generation.

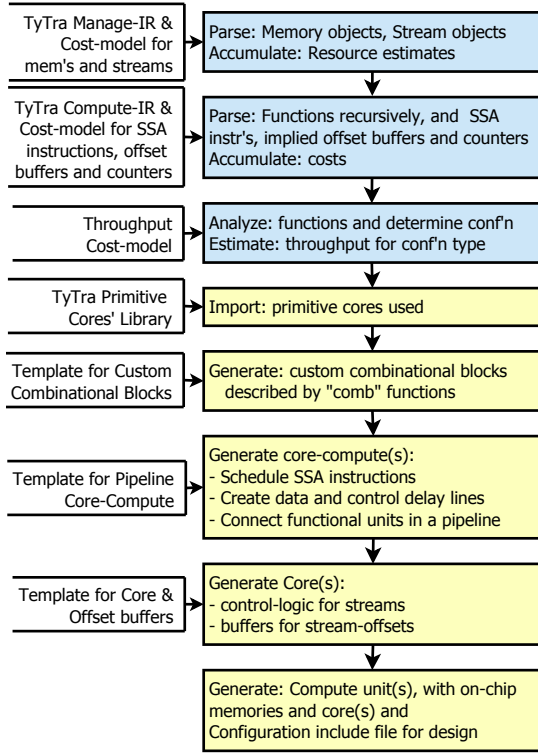


Fig. 3. The TyTra back-end compiler flow, showing the estimation flow (blue/first three stages) and code generation flow (yellow). This flow is subset of the larger flow in Figure 1.

Our throughput performance measure is *EWGT* (*Effective Work-Group Throughput*), defined as the number of times an entire work-group (i.e. all work-items) executes the kernel every second. Following is the generic expression which applies to the entire design space and expressions for configurations of interest can be derived from it.

$$EWGT = \frac{L \cdot D_V}{N_R \cdot \{T_R + N_I \cdot N_{to} \cdot T \cdot (P + I)\}}$$

Where: *EWGT* = Effective Workgroup Throughput; *L* = Number of parallel lanes or threads on the FPGA; *D<sub>V</sub>* =

Degree of vectorization per lane; *N<sub>R</sub>* = Number of FPGA configurations needed to execute the complete kernel for the entire work-group; *T<sub>R</sub>* = Time taken to reconfigure FPGA; *N<sub>I</sub>* = Number of instructions per Processing Element<sup>3</sup>; *N<sub>TO</sub>* = Ticks taken by one instruction; *T* = FPGA clock period; *P* = Pipeline depth; *I* = Number of work-items. This generic expression can be reduced based on the FPGA configuration. For the typical case of using an FPGA for HPC applications where the FPGA implements one or more pipeline lanes of the kernel, *D<sub>V</sub>*, *N<sub>R</sub>* and *N<sub>I</sub>* would all reduce to 1.

For estimating resource utilization, we observed that the regularity of FPGA fabric allows simple cost expressions for most instructions. These are then used by our compiler to evaluate overall costs for the design. A certain amount of uncertainty is introduced as our models do not take into account the optimizations done by the synthesis tools, but the estimate remain accurate enough to achieve the purpose of making design choices. The novelty here is that through a well-defined syntax at a low abstraction, the TyTra-IR exposes the parameters in the cost-model expressions, which can be extracted by our compiler. If we were to use a higher-level language as an internal IR to represent the design variants, a more thorough and time-consuming build would be required (as used by e.g. the Maxeler tool flow [8]), which is not suitable for comparing a large number of variants.

The code-generator creates a datapath architecture on the FPGA, with a pipeline of primitives as well as customized functional units, and exposes ILP by scheduling operations in parallel where possible. Streams are automatically created that connect these one or more pipelines to on-chip memories data for the streams. Our preliminary work on the cost model and code-generator is limited to working with on-chip memories of the FPGA.

#### V. USING THE COMPILER ON THE SOR EXAMPLE

For proof-of-concept of our cost model and prototype back-end compiler, we hand-coded in the TyTra-IR some design variants of the SOR kernel as discussed in §II. Figure 4 shows the translation of the SOR kernel to TyTra-IR configured as a single pipeline. The Manage-IR which declares the memory and stream objects is not shown. Note the creation of offsets of input stream *p* in lines 6–9, which create streams for the six neighbouring elements of *p*. These offset streams, together with the input streams shown in lines 2–4 form the *input tuple* referred to in §II-C. This tuple is fed into the datapath pipeline described in lines 10–15. Figure 5 shows the kernel’s realization as a pipeline as described by this code.

The same SOR example can be expressed in the IR to represent *thread-parallelism* by adding multiple lanes, corresponding to a reshaped data along 4 rows, as shown in Figure 6. This is one of the many possible variants generated in the high-level Idris code through type transformations, as described in §II-E.

##### A. Evaluating TyTra-IR Design Variants using the Cost-Model

Figure 7 shows evaluation of variants generated by reshaping the input streams – implying a different configuration on

<sup>3</sup>For pipelined cores on the FPGA, a PE is always configured for one instruction.



```

1  ; **** COMPUTE-IR ****
2  @main.p = addrSpace(12) ui18,
3          !"istream", !"CONT", !0, !"strobj_p"
4  ;...[more inputs]...
5  define void @f0(...args...) pipe {
6      ;stream offsets
7      ui18 %pip1=ui18 %p, !offset, !+1
8      ui18 %pkn1=ui18 %p, !offset, !-ND1*ND2
9      ;...[more stream offsets]...
10     ;datapath instructions
11     ui18 %1 = mul ui18 %p_i_p1, %cn21
12     ui18 %2 = mul ui18 %p_i_n1, %cn2s
13     ;...[more instructions]...
14     ;reduction operation on global variable
15     ui18 @sorErrAcc=add ui18 %sorErr, %sorErrAcc
16 }
17 define void @main () {
18     call @f0(..args...) pipe }

```

Fig. 4. Abbreviated TyTra-IR code for the SOR kernel configured as a single pipeline lane.

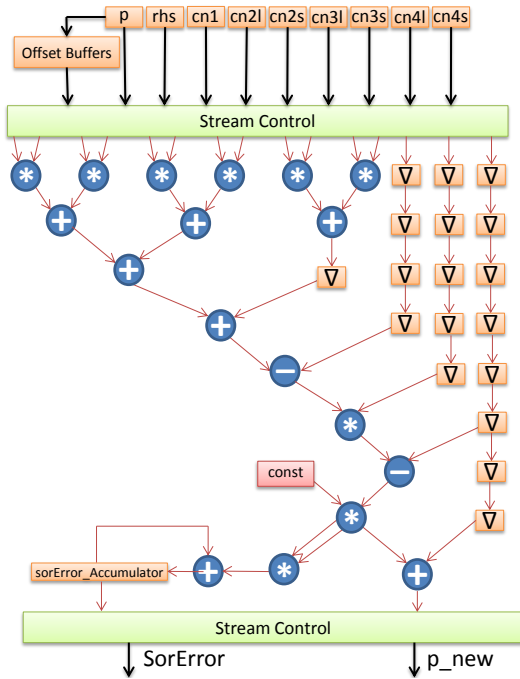


Fig. 5. Illustration of the pipelined dataflow of the SOR kernel generated by our compiler. Only pass-through pipeline buffers are shown; all functional units have pipeline buffers as well. The blocks at the top refer to on-chip memory for each data.

the FPGA – and costing the corresponding IR description. If data is transported between the CPU and device every time a new call to SOR is made, then beyond 4 lanes, we encounter the *host communication wall*; any increase in number of lanes will not improve performance unless the communication-to-computation ratio decreases by doing more kernel iterations per invocation of SOR. Alternatively, if the all the data is made available in the device's global memory, i.e. the DRAM on the device board, then the communication wall moves to about 16 lanes. We encounter the *computation-wall* when we cross six lanes, where we run out of LUTs on the FPGA. However, we can see other resources are hugely underutilized, and some sort of resource-balancing can lead to further performance improvement.

```

1  ; **** COMPUTE-IR ****
2  @main.p0 = addrSpace(12) ui18,
3          !"istream", !"CONT", !0, !"strobj_p"
4  @main.p1 = ...
5  @main.p2 = ...
6  @main.p3 = ...
7  ;...[other inputs]...
8  define void @f0(...args...) pipe {...}
9  define void @f1 (...args...) par {
10     call @f0(...args...) pipe
11     call @f0(...args...) pipe
12     call @f0(...args...) pipe
13     call @f0(...args...) pipe }
14 define void @main () {
15     call @f1(..args...) par }

```

Fig. 6. Abbreviated TyTra-IR code for the SOR kernel configured with multiple pipelines lanes corresponding to reshaped data.

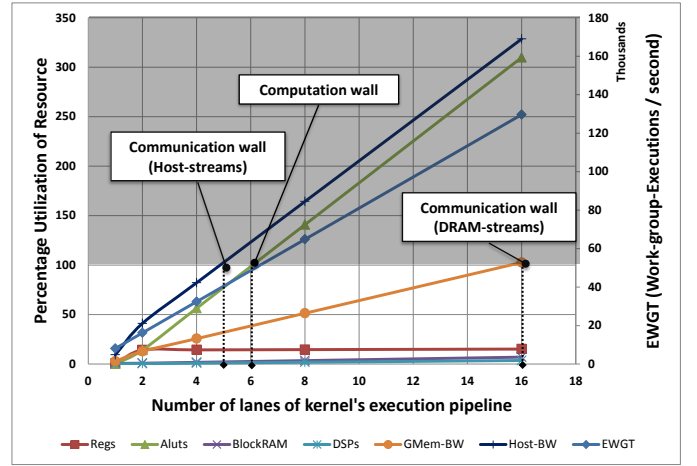


Fig. 7. Evaluation of variants for the SOR kernel generated by changing the number of kernel-pipelines. There are 16 data points for pressure along each dimension. We assume the kernel repeats 10 times to converge.

We can also evaluate the impact of re-using data read from the host into the device on-chip memory. This re-use effects the extent to which we are exploiting the hardware parallelism of the configuration on the FPGA. As shown in *Host-IO* series of Figure 8, if the SOR kernel is repeated less than 16 times, we are in the *IO-bound* zone, not fully utilizing the eight lanes in the design. Further increase in the repetition of kernel brings us into the *computation-bound* zone, where we can get better performance by optimizing the design to use lesser or more balanced resources, or possibly by moving part of the kernel to a peer device. This transition from IO to compute bound performance happens at a smaller lane-count IO is from the device DRAM with a much higher bandwidth.

We have illustrated here how the TyBEC estimator can be used to: evaluate many design variants and the trade-offs involved, generate feedback for optimizations, and achieve a near-optimal design point. We would like to highlight that the estimator is very light-weight, and e.g. the evaluation of the five design variants in Figure 7 takes a few seconds. It is orders-of-magnitude quicker than e.g. the Maxeler flow that takes tens of minutes to give preliminary resource estimates for one variant.

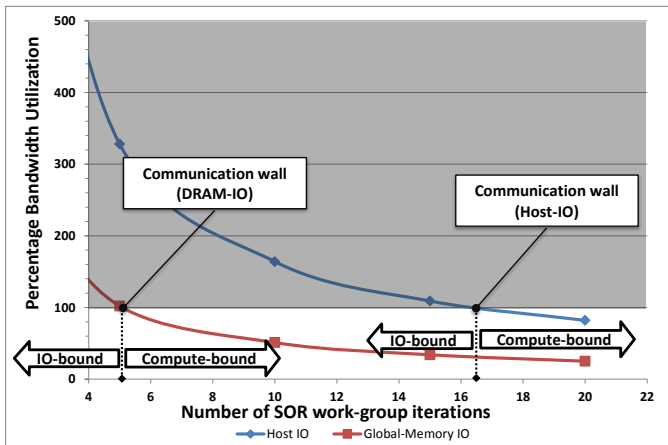


Fig. 8. Evaluating the effect of number of work-group iterations, on whether performance is IO or compute bound, for both host and DRAM IO scenarios (pipeline lanes on FPGA fixed at 8).

Resource	1-lane (E)	1-lane (G)	4-lane (E)	4-lane (G)
ALUTs	239	164	148K	146K
REGs	725	572	76,628	77,260
BRAM(bits)	186K	186K	449K	682K
DSPs	9	12	36	24
Cycles/Kernel	1,746	1,742	436	446
EWGT	190K	222K	763K	488K

TABLE I. COST AND THROUGHPUT ESTIMATED FROM IR (E), COMPARED WITH RESULTS FROM GENERATED (G) VERILOG CODE SYNTHESIZED FOR A STRATIX-5 DEVICE, FOR 1-LANE AND 4-LANE VARIANTS OF SOR KERNEL.

### B. Performance of the Estimator

To evaluate the accuracy of our cost model, we used it on two design variants. We then simulated and synthesized the equivalent HDL code. Table I shows their comparison. There is some difference in the EWGT estimate due to deviation in underlying frequency estimate, but it can be seen that the cycles/kernel estimate is much more accurate. These results confirm our observation that an IR designed at an appropriate abstraction will allow fast estimates of cost and performance that are accurate enough to make design decisions.

## VI. RELATED WORK

The use of type-transformations to generate design variants for FPGAs the way we have described in this paper is, to the best of our knowledge, an entirely original contribution, and the key novelty of our work. However, there is considerable related work from the perspective of TyTra-flow being a high-level programming tool for FPGAs. MaxJ is a Java-based custom language used to program FPGAs [8]. Our IR has been informed by a study of the MaxJ language. However TyTra-IR and MaxJ are at entirely different abstractions, with the latter positioned to provide a programmer-friendly way to program FPGAs. The TyTra-IR, being a compiler target, is fine-grained and at a lower abstraction, allowing a much better observability and controllability of the design. It also enables a light-weight cost model.

Altera-OCL is an OpenCL compatible development environment for targeting Altera FPGAs [5]. A comparison of OpenCL with TyTra-IR would come to similar conclusions as arrived in relation to MaxJ. In addition, we feel that the

intrinsic parallelism model of OpenCL, which is based on multi-threaded work-items, is not suitable for FPGA targets which offer the best performance via the use of deep, custom pipelines.

## VII. CONCLUSION

We have shown our approach to automated exploration of the design space of FPGA, and generating HDL code for programming the chosen design variant. Starting with a high-level functional language *Idris*, we demonstrated our method of creating program variants through the use of higher-order functions and type-transformations. These program variants map to design variants for the FPGA, expressed in our custom IR language, the TyTra-IR. It not only allows us to describe multiple design variants for the same problem, but also to directly associate each variant with a cost for cost-driven optimization. Using a Successive Over-Relaxation kernel taken from a real-world weather simulator as an example, we illustrated the generation of program variants using type-transformations in *Idris*, expressing design variants in the TyTra-IR, and costing different variants to evaluate trade-offs. We demonstrated the accuracy of the cost-model by comparing against synthesis figures. Our future work will look at automating the generation of variants and translation to IR from *Idris*, and improving the sophistication of the cost model for – among other things – a more generic memory model that takes into account different data access patterns.

## ACKNOWLEDGEMENT

The authors acknowledge the support of the EPSRC for the TyTra project (EP/L00058X/1).

## REFERENCES

- [1] C.-H. Moeng, "A large-eddy-simulation model for the study of planetary boundary-layer turbulence," *J. Atmos. Sci.*, vol. 41, pp. 2052–2062, 1984.
- [2] E. Brady, "Idris, a general-purpose dependently typed programming language: Design and implementation," *Journal of Functional Programming*, vol. 23, pp. 552–593, 2013.
- [3] W. Vanderbauwhede, "Inferring Program Transformations from Type Transformations for Partitioning of Ordered Sets," 2015. [Online]. Available: <http://arxiv.org/abs/1504.05372>
- [4] J. Stone, D. Gohara, and G. Shi, "Opencl: A parallel programming standard for heterogeneous computing systems," *Computing in Science Engineering*, vol. 12, no. 3, pp. 66–73, May 2010.
- [5] T. Czajkowski, U. Aydonat, D. Denisenko, J. Freeman, M. Kinsner, D. Neto, J. Wong, P. Yiannacouras, and D. Singh, "From opencl to high-performance hardware on FPGAs," in *Field Programmable Logic and Applications (FPL), 2012 22nd International Conference on*, Aug 2012, pp. 531–534.
- [6] A. Canis, J. Choi, M. Aldham, V. Zhang, A. Kammoona, J. H. Anderson, S. Brown, and T. Czajkowski, "Legup: High-level synthesis for FPGA-based processor/accelerator systems," in *Proceedings of the 19th ACM/SIGDA International Symposium on FPGAs*, ser. FPGA '11. New York, NY, USA: ACM, 2011, pp. 33–36. [Online]. Available: <http://doi.acm.org/10.1145/1950413.1950423>
- [7] S. W. Nabi and W. Vanderbauwhede, "An intermediate language and estimator for automated design space exploration on fpgas," in *International symposium on Highly Efficient Accelerators and Reconfigurable Technologies (HEART2015), Boston, USA, 2015*. [Online]. Available: <http://arxiv.org/abs/1504.045791>
- [8] O. Pell and V. Averbukh, "Maximum performance computing with dataflow engines," *Computing in Science Engineering*, vol. 14, no. 4, pp. 98–103, July 2012.